Measuring the difference in wavelength of the D lines of Sodium

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Observing the wavelength difference of the sodium emission spectrum cannot be done through the conventional use of prisms or spectrometers so interferometers must be used. In this work a Fabry–Pérot and Michaleson interferometer are used in finding the wavelength difference giving $2.34 \pm 0.002 \times 10^{-11}$ m and $5.93 \pm 0.04 \times 10^{-10}$ m respectively with the latter being consistent with the literature.

I. INTRODUCTION

The emission spectrum of sodium primarily consists of a sharp doublet with average wavelength of 5.893×10^{-7} m [1] and wavelength difference of 5.974×10^{-10} m [1] with all other fringes being much fainter. Using a prism and spectrometer set-up will not give the fine detail necessary to find the difference in wavelength as it cannot resolve to this scale. Interferometry can be used to measure the interference pattern between the two wavelengths of the doublet allowing a determination of the difference in wavelength.

In this work measurements of the wavelength difference will be made by using Fabry–Pérot and Michalson interferometers. In both cases incoming light from a sodium lamp is split into multiple beams with both interferometers using half-silvered mirrors. The path difference between the beams can be varied in both interferometers, the mirror set up in Fabry–Pérot allows for a more precise tuning of path difference. The split beams then recombine allowing the interference pattern to be observed. When the interference is at a minimum no concentric circles should be seen, changing the path difference until the next minima allows for the determination of the wavelength difference by [1]

$$\Delta\lambda = \frac{N\overline{\lambda}^2}{2d} \tag{1}$$

where $\Delta\lambda$ is the difference in wavelength, $\overline{\lambda}$ is the average value of the two wavelengths (5.893×10^{-7} m), d is the distance between the two minima and N is the number of minima counted from the first to the current minima.

II. METHOD

The sodium lamp was placed ten centimetres back from the diffusing plate so that the light was sent through the half mirrors. The interference of the split beams formed an image of concentric circles at the viewing station. Measurements of the micrometer were made when there were no dark fringes and the image was at its most blurry, this point was used as it was easier to distinguish than the point of maximum prominence of dark fringes which occurred when interference was at a maxima.

The main source of error was from trying to accurately determine the position of the minima, as judging when the image was at its most blurry was difficult. This error was reduced by finding a maximum and minimum value of distance of the minima then travelling though this range until a new maximum and minimum were found. Repeating this gave a small range of possible values for the true distance. Back-lash is the slight give in the gearing when switching direction on the micrometer screw gauge. Once the possible range was within ten units of the micrometer, rolling the screw gauge continuously in one direction until a minima was found, then repeating this and finding the average value produced a more accurate result for the distance of the minima.

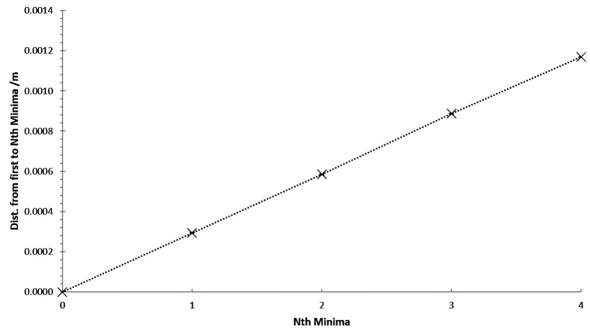

FIG. 1: The relationship of the Nth number minima and the distance from the first minima for a Michelson interferometer. The error bars are too small to be seen.

III. RESULTS

Fabry–Pérot:
The first minima was found at $1.174 \pm 0.005 \times 10^{-2}$ m and the second was found at $1.918 \pm 0.005 \times 10^{-2}$ m. Distance between the two minima was found by subtracting these values giving $7.43 \pm 0.07 \times 10^{-3}$ m. This value was then used in Eq.(1), setting N equal to one and solving for the difference in wavelength gave a value of $2.34 \pm 0.02 \times 10^{-11}$ m.

Michaelson:
As seen in Fig.1 the linear relationship by plotting Eq.(1) means that the gradient is given by

$$gradient = \frac{\overline{\lambda}^2}{2\Delta\lambda} \tag{2}$$

. As the error bars in Fig.1 are too small to be visible Least Squares Regression was used to determine the gradient of the graph which gave $2.93 \pm 0.02 \times 10^{-4}$ m. From this value and from rearranging Eq.(2) the calculated value for $\Delta\lambda$ was $5.93 \pm 0.03 \times 10^{-10}$ m.

IV. DISCUSSION

Fabry–Pérot:
The accepted value for the difference in wavelength is 5.974×10^{-10} m, the experimental value obtained was $2.34 \pm 0.02 \times 10^{-11}$ m. This result is more than three standard errors away from the accepted value in fact it is more than a factor of twenty five different.

The large inaccuracy in the result comes from only using two minima, as the exact position of the minima were uncertain. Taking only two measurements means low confidence in the final result. A potential source of error was that minima were missed when travelling between the two measured values, the number of minima in this distance is given by rearranging Eq.(1) to solve for N with $\Delta\lambda$ being the accepted value, calculating N would give a value of 25 hence if this was the case then 25 additional minima were to be found in this interval which is unlikely.

The total length of the screw gauge was 0.030 m, adding the difference in distance to the final position of the second minima we obtain a value of 0.2661 m hence there was another minima that could have been used in calculating the difference in wavelength. Note that subtracting the difference in distance from the first minima yields 0.00432 m which is too close to the start of the scale meaning confidence in the exact position of the minima would be low.

If a third measurement was taken a graph could have been plotted allowing for a more reliable result of distance between minima. This would be done if time allowed as well as retaking the reading for the original two minima. The precision of this interferometer is large however the lack of minima meant low confidence in the value of the difference between minima of leading to an inaccurate result.

Michaelson:
This interferometer gave a much more accurate result of $5.93 \pm 0.04 \times 10^{-10}$ m which lies within one standard error of the accepted value. Five different minima were measured allowing a graph to be plotted, the straight line linear relation confirms Eq.(1) and taking the gradient reduces the random error associated with determining the position of the minima.

Subtracting pairwise the distances of the minima then taking the average value gives a value of 0.000292 m. Dividing the maximum distance the mirrors can move(0.006 m) by this and subtracting the number of minima used in the experiment it can be shown that an additional fifteen minima could have been measured on this scale.

The accuracy of the result could have further been improved by using these minima if time had allowed. The gearing reduced the distance travelled by the mirror by a factor of five hence there was a larger uncertainty in a single measurement, however the ability to graph the results reduces the effect of random error leading to a more accurate result than the Fabry–Pérot interferometer albeit a less precise one.

V. CONCLUSIONS

In conclusion the difference in the wavelength of the sodium doublet has been determined though the use of two different interferometers. By measuring the distance between minima, the Fabry–Pérot interferometer gave a value of $2.34 \pm 0.002 \times 10^{-11}$ m with low confidence in the result as only two minima were used. The Michaelson interferometer gave a value of $5.93 \pm 0.04 \times 10^{-10}$ m which has lower precision but higher accuracy as more minima were sampled. This result is one standard error away from the true value.

References

[1] Absolute Determination of the Wavelengths of the Sodium D1 and D2 Lines by Using a CW Tunable Dye Laser Stabilized on Iodine, P Juncar et al 1981 Metrologia17 77

VI. ERROR APPENDIX

Fabry–Pérot:

By convention uncertainty associated with a single reading from a micrometer is taken to be one unit of the smallest division of the scale, in our case 2.5×10^{-5} m. However, it is reasonable to assume that the reading should be taken to have an uncertainty of two units due to the difficulty of determining the precise position of the minima. This gives the uncertainty for a single measurement as 5×10^{-5} m. The uncertainty in the difference in distance is given by

$$\alpha_d = \sqrt{\alpha_1^2 + \alpha_2^2} \tag{3}$$

with α_d being the uncertainty in the difference in distance, α_1 being the uncertainty in distance 1 and α_2 being the uncertainty in distance 2. [This equation and all others in the appendix are based on the error equations found in I. G. Hughes and T. P. A. Hase, *Measurements and their Uncertainties*, Oxford University Press, Oxford (2010).] The error in difference in distance is thus 7×10^{-5} m. Using the equation for functional error analysis on Eq.(1) gives

$$\alpha_{\Delta\lambda} = |\Delta\lambda - \frac{N\overline{\lambda}^2}{2(d + \alpha_d)}| \tag{4}$$

where $\alpha_{\Delta\lambda}$ is the error in the difference in wavelength. The total error in the difference in wavelength is thus 2×10^{-13} m

Michaelson:

The error of a single reading of the micrometre was 2×10^{-6} m as it was scaled down by a factor of five because of the gearing. The gradient of the line of best fit from Fig.1 has an associated error from Least Squares Regression. Using the LINEST function in Excel this value was given as 2×10^{-6} m, which coincidently is the same as a single reading. Performing functional error analysis with Eq.(2) gives the equation

$$\alpha_{\Delta\lambda} = |\Delta\lambda - \frac{N\overline{\lambda}^2}{2(gradient + \alpha_g)}| \tag{5}$$

where α_g is the error in the gradient given by LINEST. Using this gives the error in difference in wavelength as 4×10^{-12} m.