Measuring the Planck constant

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In this report Bragg's Law and the Duane-Hunt law will be used to determine a value of Planck's constant. A LEYBOLD 554800 X-ray apparatus will generate bremsstrahlung curves for different voltages and through data analysis the minimum wavelength of the X-rays can be obtained. From this data the experimental value of Planck's constant is calculated to be $6.4 \pm 0.5 \times 10^{34}$ Js which is within one standard error of the true value.

I. INTRODUCTION AND THEORY

During the early 20th century the discovery of X-rays by Wilhelm Röntgen lead to the field of crystallography being developed. Further study of X-rays lead to Bragg's law and the Dune-Hunt law being formulated. From these two laws it was then possible to determine a value of Planck's constant.

In this experiment X-Rays are produced by accelerating electrons through a high voltage and colliding them with a molybdenum plate, as the electrons decelerate, they emit bremsstrahlung radiation (breaking radiation) along a continuous range of wavelengths. This radiation is incident onto a plate of sodium chloride crystal on a rotating mount so the angle of incidence can be varied. The crystal allows only certain wavelengths to reach the detector, which are given by Bragg's law:

$$\lambda = 2dsin(\theta) \tag{1}$$

with λ being the wavelength of incoming light, θ being the angle of incidence and d being the distance between the sodium and chlorine atoms in the crystal, which is 0.282 nm [1] (Bragg's law describes a wave like property of light). The wavelength at which counts are first detected is called the minimum wavelength. The maximum energy of the photon is that of the electron that created it (this is a particle like property), from this the Duane-Hunt law can be derived:

$$\lambda_{min} = \frac{hc}{eV} \tag{2}$$

with λ_{min} being the minimum wavelength, h being Planck's constant, c being the speed of light in a vacuum(299800000 ms^{-1} [2]), e being the elementary charge constant (1.602×10^{-19} C [2]) and V being the accelerating voltage.

II. METHOD

In this experiment a LEYBOLD 554800 X-ray apparatus was used. The devise was electronically controlled and was set up to produce bremsstrahlung curves through the process described in the theory section. Before the data was collected for a given voltage a quick scan was performed over a large angle range so that angle limits could be set such that the detailed scan occurred across the linear region of each curve. The highest input current was used (1 mA), this increased the number of X-rays produced and hence detected in a given time reducing the random error in counting. The most precise plot was formed by using the smallest angle division (0.1°) and the scanning angle was converted to wavelength through Bragg's law Eq.(1). Three repeats of the data were taken with each scan taking seven minutes, this allowed enough varied data to be collected in the time given, background count rate was also recorded and averaged. Using the linear region to determine the minimum wavelength, Planck's constant can be found by plotting this against one over the accelerating voltage and using the Duane-Hunt law. Most of the error analysis is done during the data analysis and results section.

III. DATA ANALYSIS AND RESULTS

Taking the average of the three count rates for each of the voltages and plotting against the incident wavelength gives the bremsstrahlung curves shown in Fig.(1), the error bars have been removed for clarity. The linear region shown in

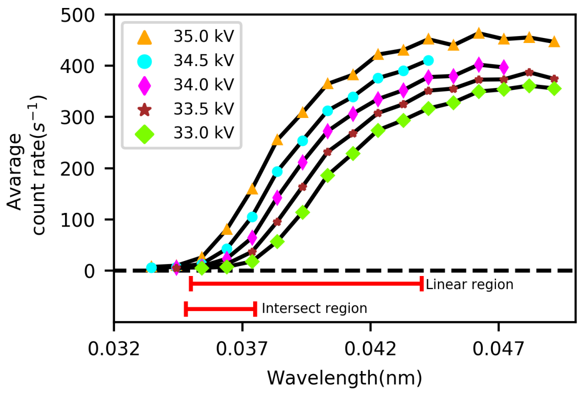

FIG. 1: Bremsstrahlung curves for different voltages

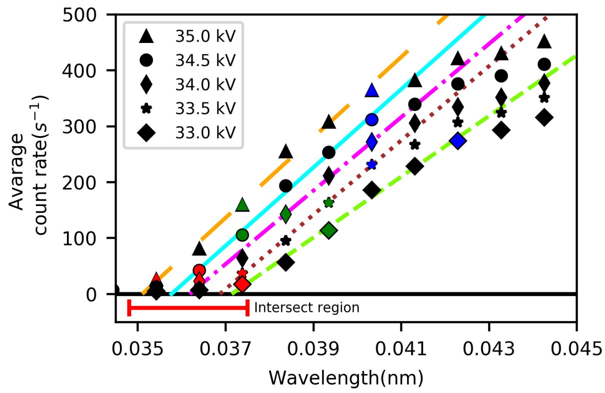

FIG. 2: Linear region of each bremsstrahlung curve

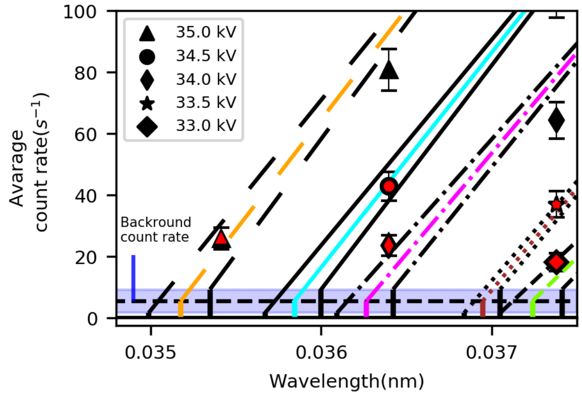

FIG. 3: Intersection of lines of best fit and background count rate

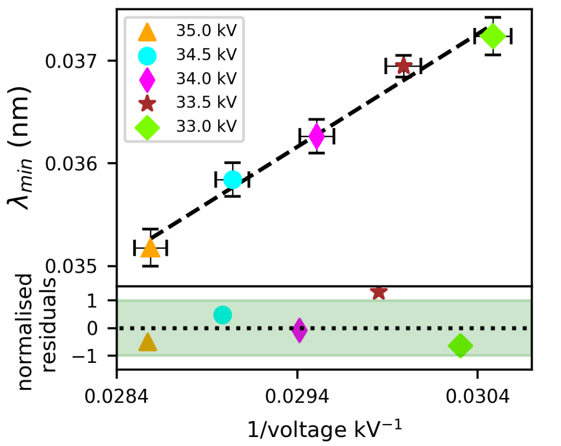

FIG. 4: Using the Duane-Hunt law to find Plank's constant

Fig.(2) was found by considering three distinct points on each curve. The red point on each curve indicates that start of the linear region, this is the first point that is two standard errors away from the background line (this shows significant deviation from the background). The green point approximates the point of inflection of the curve which is where the graph has zero curvature and hence is linear. This was calculated by considering the gradient of a specified point:

$$f'(x_i) = \frac{R_{i+1} - R_{i-1}}{\lambda_{i+1} - \lambda_{i-1}} \tag{3}$$

With x_i being the specified point, i denoting the index of the point, R denotes the count rate of a point and λ is the wavelength of a point. The green point is the one that gives a maximum value of $f'(x_i)$ as this approximates the point of inflection. The blue point represents the end of the linear region of the curve. This point maximises the size of the linear region whist maintaining a high correlation. Considering each point after the green point and applying

$$g(x_i) = r_i^{\alpha} s \tag{4}$$

with s being the number of points considered after the green point and r_i^{α} being the product moment correlation coefficient raised the the power α with r_i given by

$$r_i = \frac{\Sigma\lambda_n R_n - \frac{\Sigma\lambda_n \Sigma R_n}{n}}{\sqrt{[\Sigma\lambda_n^2 - \frac{(\Sigma\lambda_n)^2}{n}][\Sigma R_n^2 - \frac{(\Sigma R_n)^2}{n}]}} \tag{5}$$

,with the sum going from the green point to the ith point. This is a measure of correlation between 0 and 1 with α being required so that it is costly if a new point decreases correlation (a good value was found to be 400 by trial and error). The point that gives the largest value of this is the blue point. Best fit lines are then formed by performing least squares regression on the red to blue region.

Fig.(3) shows the intersect between the lines found in Fig.(2) and the average background count rate given by $5 \pm 4\text{s}^{-1}$. The uncertainty in least squares regression from Fig.(2) forms two further lines for each voltage. The wavelengths at which these intersections occur gives values and uncertainties for the minimum wavelength for each voltage.

From the values of the minimum wavelength found in Fig.(3) plotting these against one over voltage yields Fig.(4). Using least squares regression, the gradient is given as 1.21 ± 0.09 kVs^{-1}. Using this and rearanging Eq.(2) we get a value of Planck's constant as $6.4 \pm 0.5 \times 10^{-34}$ Js.

IV. DISCUSSION

The true value of Planck's constant is 6.63×10^{-34} Js [2], the experimental value was $6.4 \pm 0.5 \times 10^{-34}$ Js which is within one standard error of the true value giving a good agreement, this is a reliable result as in Fig.(4) four out of five residuals lie within one standard error of the gradient. The effect of random error in counting has been reduced by averaging counts, extrapolating lines to the background count rate and using a high current. Human error has been reduced by developing an algorithm to find the linear region of the curves, although α had to be found by trial and error no bias is introduced as the same rules apply to each curve. The experiment could most be improved by using equipment with higher precision in angle and taking more measurements around the ends of the linear region, this would improve precision of the gradient hence lower the error of Planck's constant. The analysis could be improved by applying a weighted regression when finding best fit lines as the error in count rate is non-linear. Using weighted regression would also account for error in voltage and angle, instead of just using the error given by least squares regression.

V. CONCLUSIONS

In this experiment Planck's constant was determined by using a computer-controlled apparatus to generate data for bremsstrahlung curves at different voltages. Using data analysis the minimum wavelength was found at each voltage. Planck's constant was then found by the Duane-Hunt law giving an experimental value of $6.4 \pm 0.5 \times 10^{-34}$ Js which is within one standard error of the true value. This could be improved with more precise angle measurement and using weighted regression during analysis.

References

[1] "X-ray Transition Energies". NIST Physical Measurement Laboratory.

[2] "The 2018 CODATA Recommended Values of the Fundamental Physical Constants". NIST Physical Measurement Laboratory.

VI. ERROR APPENDIX

Errors in this experiment include the limited precision of the X-ray apparatus, having error on the angle and voltage of one unit of the smallest division of the screen being $0.1°$ and 0.1 kV respectively. The constants of c and e are taken to have zero error and are only shown to four significant figures (as this is more than enough for the precision of the experiment) and for d the error is too small to be considered. The main source of error in the experiment was the number of counts detected, this can be modelled as a Poisson distribution with the uncertainty given by $\sqrt{N}$,where N is the number of counts [This equation and all the others in the appendix are from the error equations found in I.G Hughes and T.P.A Hase, Measurements and their Uncertainties, Oxford University press, Oxford (2010)]. To convert this to error in count rate we use a functional error approach

$$\alpha_R = |\frac{1}{N} - \frac{1}{N+\sqrt{N}}| \quad (6)$$

, this method is also used to determine the uncertainty in the background count rate. The final error in Planck's constant is found by again using a functional error approach

$$\alpha_h = |\frac{e \times gradient}{c} - \frac{e \times (gradient + \alpha_{gradient})}{c}| \quad (7)$$

, with the error in gradient given by the LINEST function of Excel.