The Surprising Phase Transitions in Schools of Fish

Investigating the regimes and vortex formation of the Vicsek model

Robert Smith
Durham University
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Supervised by Dr Steven Wrathmall

Outline

- Background and Theory
- Live Demonstration
- Recap of Main Results
- Computational Speed-Up
- Vicsek Modification
- Vortex Detection
- Future Work

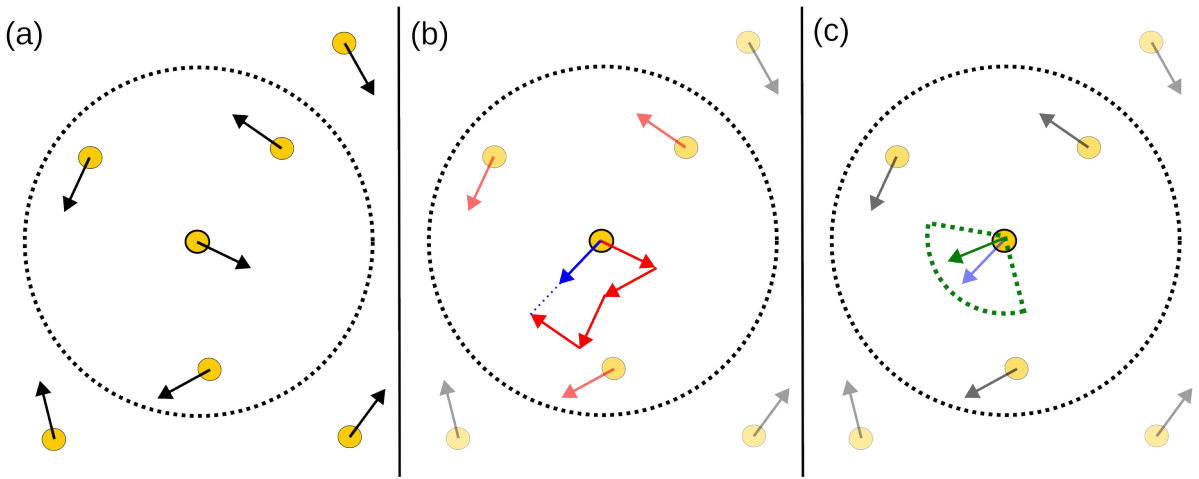
Main objective

Visualise the different regimes of the Vicsek model

Background/Theory

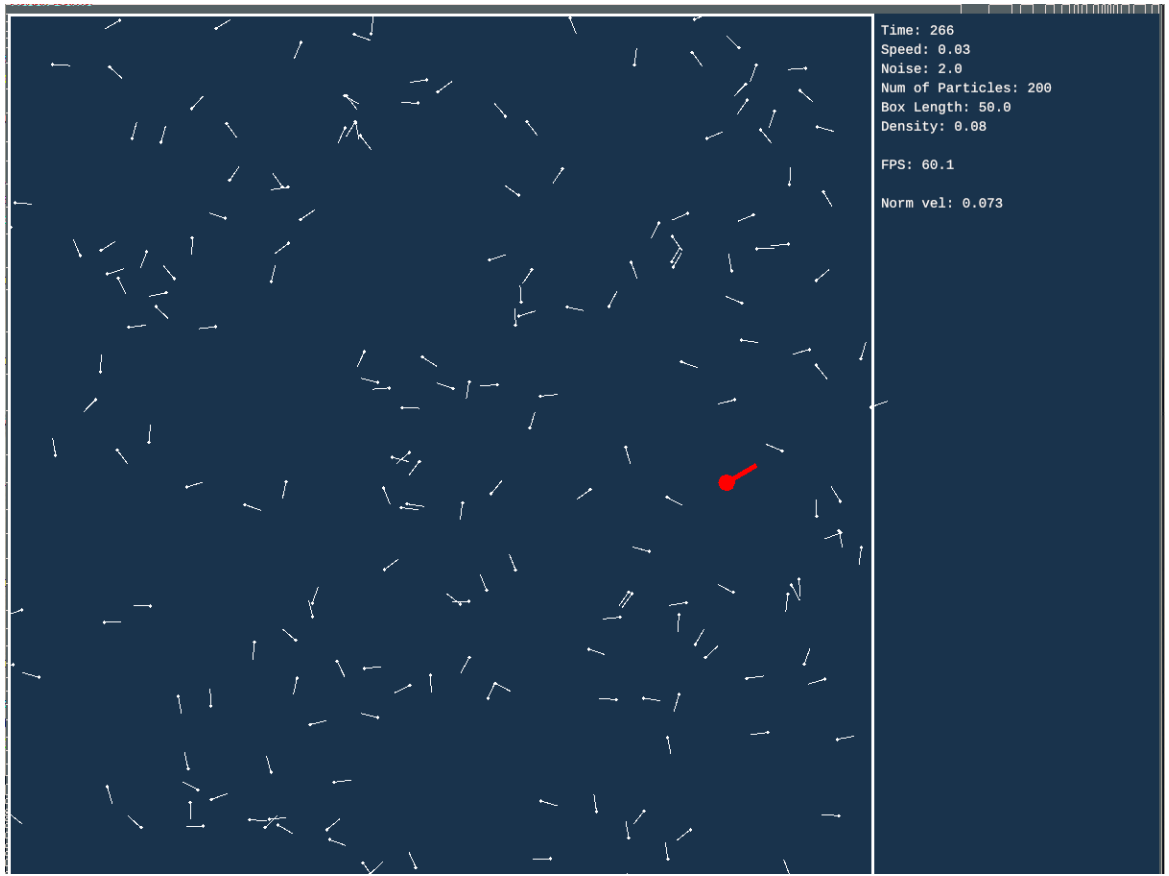
$$\boldsymbol{x}_i(t+1) = \boldsymbol{x}_i(t) + \boldsymbol{v}_i(t)\Delta t \qquad (1)$$

$$\theta(t+1) = \langle\theta(t)\rangle_r + \Delta\theta \qquad (2)$$



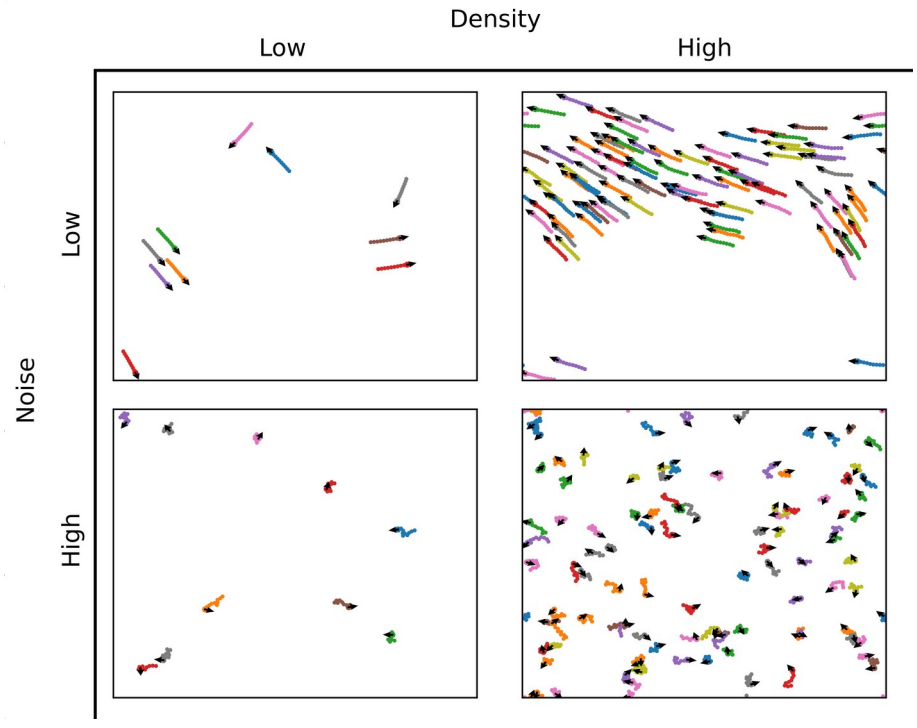
T. Vicsek, A. Czir´ok, E. Ben-Jacob, I. Cohen, and O. Shochet, “Novel type of phase transition in a system of self-driven particles,” Physical Review Letters, vol. 75, no. 6, pp. 1226–1229, Aug. 7, 1995, Publisher: American Physical Society. DOI: 10.1103/PhysRevLett.75.1226. [On-line]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.75.1226 (visited on 09/27/2022).

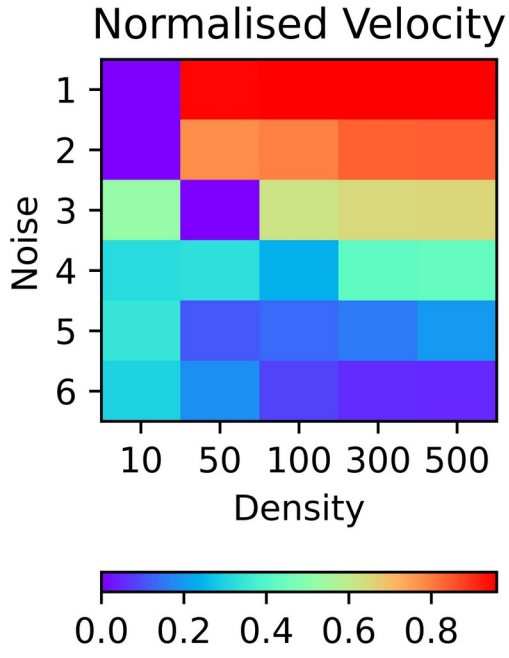
Live Demonstration



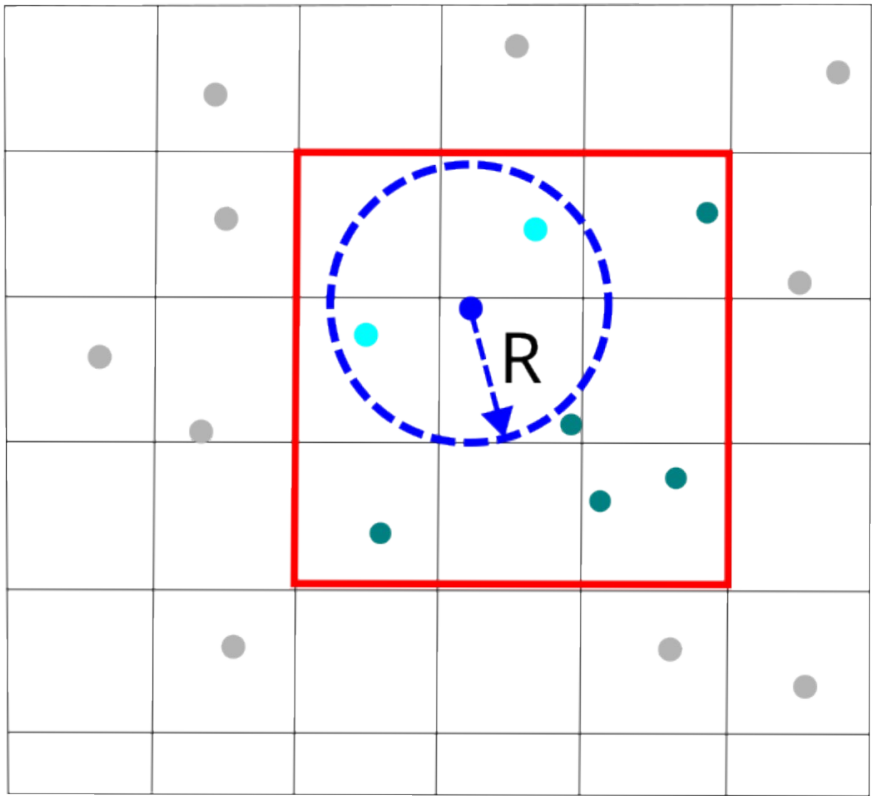
https://github.com/RobSmith2000/VicsekSimulation

Results


Density
Low
High
Noise
Low
High


Normalised Velocity
Noise
1
2
3
4
5
6
10
50
100
300
500
Density
0.0
0.2
0.4
0.6
0.8

Computational Speed Up



In Python: 1000 particles, 30 steps, 1 minute

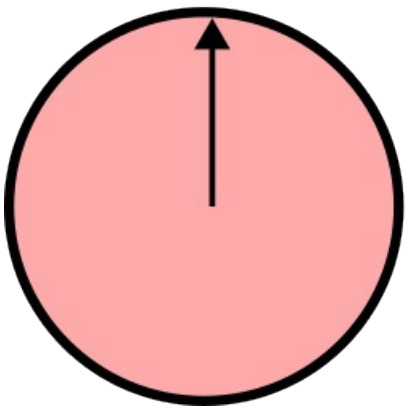
In Rust: 10000 particles, 30 steps, 1 second

$$[3, 2], [0, 3], [1, 2], [1, 4]...$$

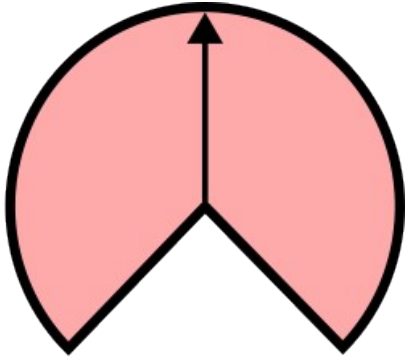
CPU 1 CPU 2 ...

Find 3x3 grid and update velocity

Vicsek Modification: Blind Angle



$\phi = 0$



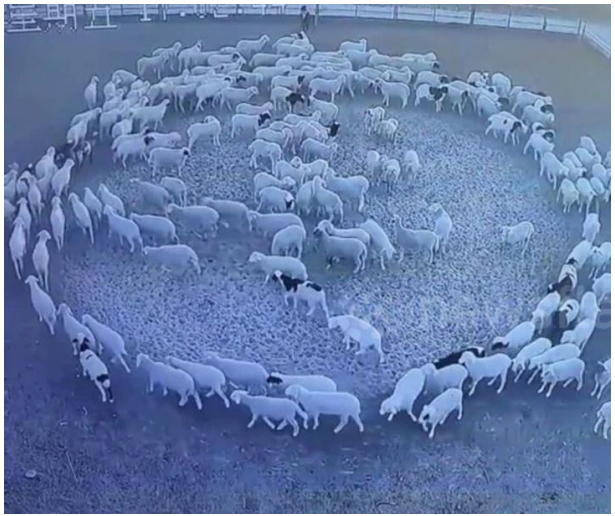
$\phi = 60$

$$\theta(t+1) = \begin{cases} \langle \theta(t) \rangle_{r,\phi} + \Delta\theta & \text{for } |\Delta\Theta| < \omega \\ \theta(t) + \omega + \Delta\theta & \text{for } \Delta\Theta \geq \omega \\ \theta(t) - \omega + \Delta\theta & \text{for } \Delta\Theta \leq -\omega \end{cases}$$

A. Costanzo and C. K. Hemelrijk, “Spontaneous emergence of milling (vortex state) in a vicsek-like model,” Journal of Physics D: Applied Physics, vol. 51, no. 13, p. 134 004, Mar. 2018. DOI:10.1088/1361-6463/aab0d4. [Online].
Available: https://dx.doi.org/10.1088/1361-6463/aab0d4

Vicsek Modification: Milling Demonstration

https://www.pond5.com/stock-footage/item/8568159-ants-spiral-death

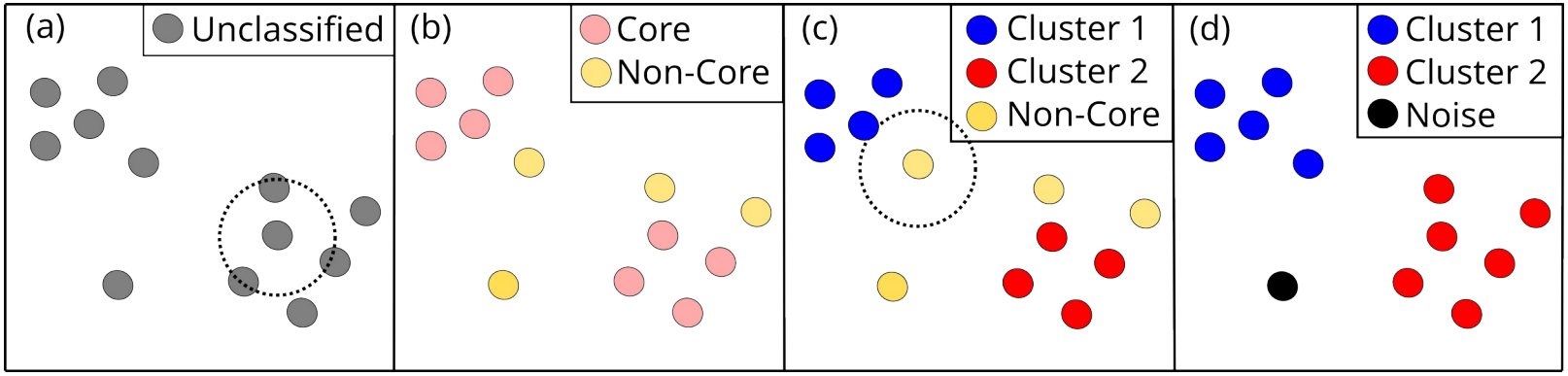


https://im.indiatimes.in/content/2022/Nov/Sheep-herd_6378ba80c383a.jpg?w=725&h=608&cc=1

https://www.pinterest.com/pin/57843176439961577/

Vortex Detection: Vortex Criterion

DBSCAN



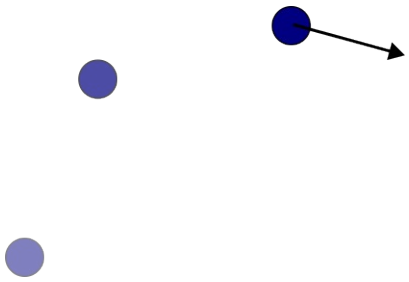
$$v_a = \frac{1}{N}\left|\sum_{i=1}^{N}\mathbf{u}_i\right| \qquad m_a = \frac{1}{N}\sum_{i=1}^{N}\frac{|\mathbf{r}_{\mathrm{com},i}\times\mathbf{u}_i|}{|\mathbf{r}_{\mathrm{com},i}|}$$

Vortex Criterion

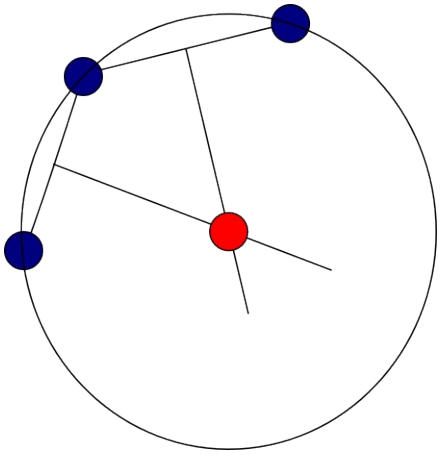
$$v_a < 0.5$$
$$m_a > 0.7$$

D. Deng, “Dbscan clustering algorithm based on density,” in 2020 7th International Forum on Electrical Engineering 2020, pp. 949–953. DOI: 10.1109/IFEEA51475.2020.00199

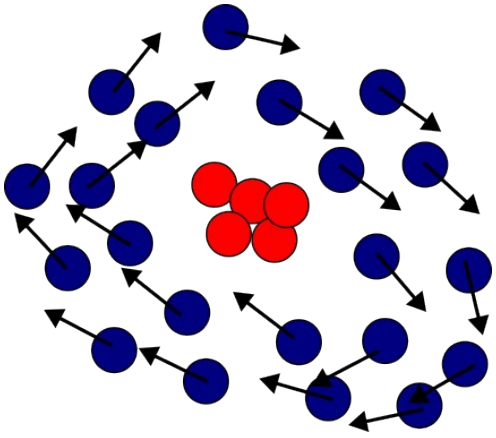
Vortex Detection: Improved Algorithm



(1)



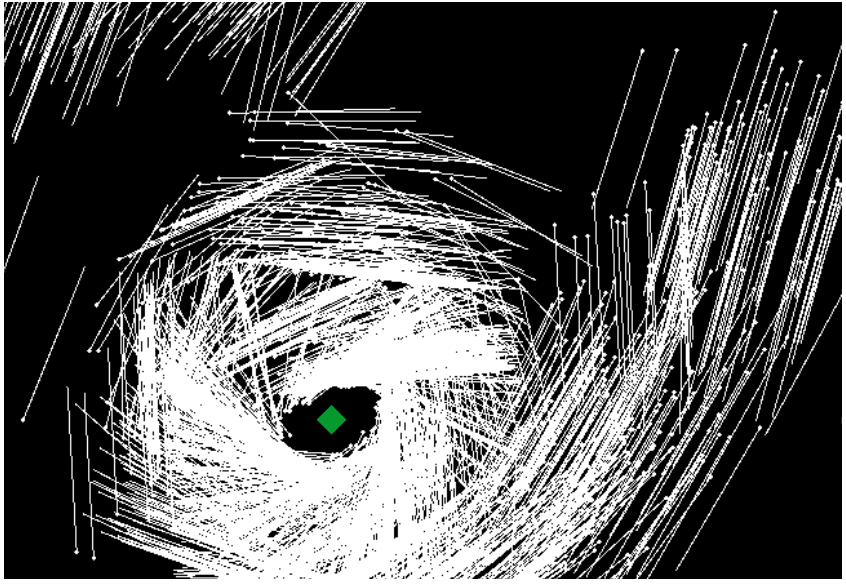
(2)



(3)

Future Work

- Study the nature of the vortex transition
- Comparison of vortex detection methods



- Curved arrow representing the flow of the presentation, have an ability to quickly move through slides at the end instead of clicking many times